Investigating Chaos in Time Series: Evidence from the Cryptocurrency Market

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Outline

Stochastic vs. Chaos

- The dynamic of a time series (or a system) could be of two types:
  - Stochastic
  - Deterministic
- Determining the characterization is crucial to understand and properly model the series.
- Most of analytical tools are designed for stochastic time series.
- A dynamic system is said to be chaotic if it has one of the following properties:
  - deteministic
  - limited
  - aperiodic
  - sensitive to initial conditions

Measure of Chaos

- Lyapunov exponent gives a quantitative measure of the sensitivity to initial conditions.
- It reflects the average rate of separation (expansion or contraction) per iteration of two trajectories on an attractor with very close initial conditions.
- Given a series x the Lyapunov exponent λ could be approached via : $|\delta x(t)| \approx e^{\lambda t}|\delta x(0)|$
- We usually compute the Maximum Characteristic Lyapunov Exponent (MCLE)

$$\lambda = \lim_{n \to \infty} \lim_{\delta \to 0} \frac{1}{n} ln(\frac{||x_n - x_n^{'}||}{||x_0 - x_0^{'}||})$$

  - $\lambda < 0$: trajectories converge (no chaos)
  - $\lambda = 0$: no contraction or expansion
  - $\lambda > 0$: trajectories diverge (chaos)

Measure of Chaos

- Multidimensional dynamic systems have a multitude of Lyapunov exponents.
- In practice, we test the **Maximum Lyapunov exponent** to determine the nature of the system.
- Other tools may be used as the *correlation dimension* or the study of *attractors*.

- Fractal analysis stems from earlier works of Benoit Mandelbrot on cotton prices in 1950's.
- As a geometric method, it studies irregular shapes by computing the *fractal dimension* D which is a proxy of an object's complexity.
- In time series analysis, we usually compute the *Hurst exponent* H via Rescaled Range Analysis (R/S) which measures the long-term memory, then deduce the fractal dimension via $D = 2 - H$ where $0 < H < 1$
  - $0 < H < \frac{1}{2}$: anti-persistence (mean-reverting)
  - $H = \frac{1}{2}$: memoryless process (Brownian motion)
  - $\frac{1}{2} < H < 1$: persistence (long-memory)

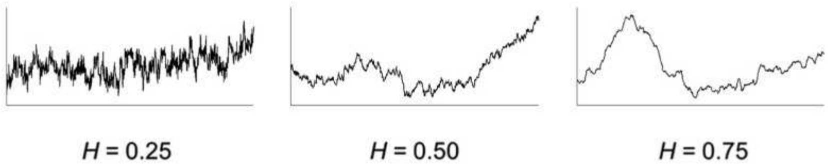

Figure: Simulated fractional Brownian motion for different values of H

- Researchers pointed out that most financial time series exhibit changing regimes and unstable statistical properties: *stylized facts*.
- This led to abandon the hypothesis of a single fractal dimension (scale invariance), and to consider a set of fractal dimensions known as multifractal analysis (scale variance).
- This can help interpreting the memory affecting the type of small/large fluctuations and determining to richness of this variability (mutlifractal spectrum).
- We use the Multifractal Detrendend Fluctuation Analysis (MF-DFA) to investigate the changes of local Hurst exponents H_q with the given set of exponents q.

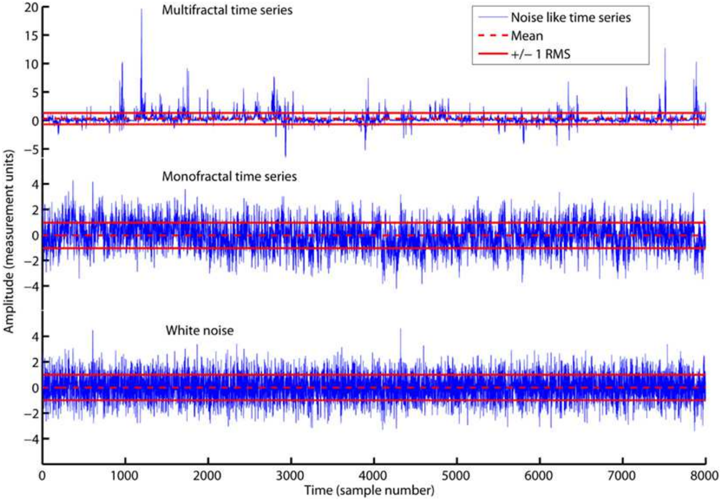

Figure: Examples of generated time series. (Source : Ihlen, 2012)

Case Study

- We study the cryptocurrency market and investigates its underlying/embedded patterns.
- " **Cryptocurrency** *is digital asset designed to work as a medium of exchange that uses strong cryptography to secure financial transactions, control the creation of additional units, and verify the transfer of assets*" (Wikipedia).
- This *fintech* innovation relies on " *decentralized control of each cryptocurrency, working through distributed ledger technology, typically a blockchain, that serves as a public financial transaction database* "
- Ongoing (fierce) debate between supporters and opponents of these virtual currencies.
- Question: Is the cryptocurrency market really safe ? well-valued ?

Figure: Some cryptocurrecies' symbols. Source:
https://www.finder.com.au/cryptocurrency-list-all

- We analyze the 5 most cryptocurrencies by rank (market capitalization), from April 2013 to July 2018.
  - Bitcoin
  - Bitcoin Cash
  - EOS
  - Ethereum
  - XRP
- Daily time series data fetched using the **crypto** package with the help of **dplyr** package for data manipulation/extraction.
- Closing prices are used for each currency.
- An emphasis will be given to the Bitcoin as it demonstrates the highest volatility and price fluctuations.
- R packages used: **pracma**, **MFDFA**, **fNonlinear**, **tseriesChaos**, **nonlinearTseries**

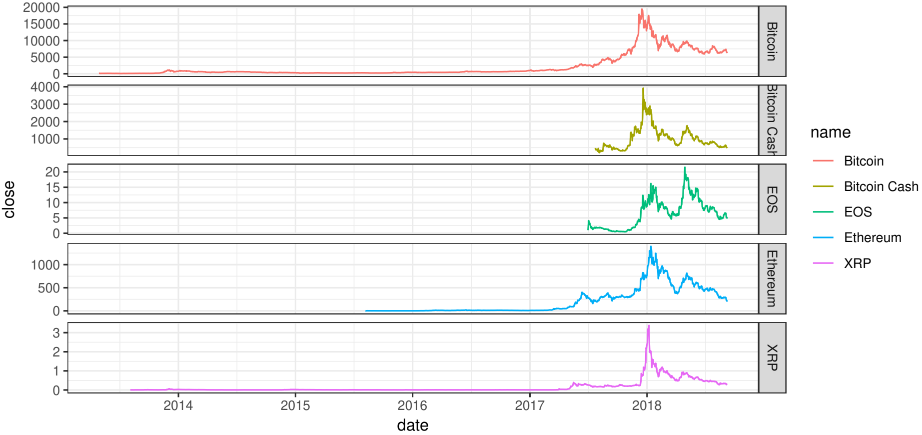

Figure: Close prices of the top 5 currencies by rank.

Estimations

Series	Length	Max. Lyapunov	Hurst exponent
Bitcoin	1,960	0.0327	1.004
Bitcoin Cash	413	0.0786	0.856
EOS	435	0.0773	0.975
Ethereum	1,129	0.0160	0.991
XRP	1,862	0.0100	0.888

MF-DFA

- Using the **mfdfa** package on the Bitcoin closing prices series (log returns) we notice a relative persistent behavior of both small or large fluctuations.
- Small fluctuations are highly persistent (long memory) while large fluctuations remain slightly persistent.
- Hence, the series could be seen as non-stationary, generated by a complex process, confirming the chaotic pattern hypothesis.

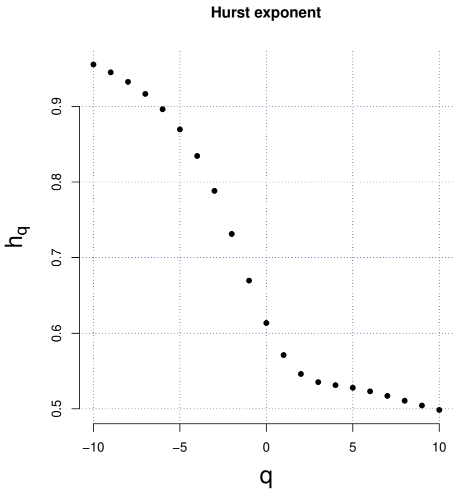

Figure: Local Hurst exponents (log returns of the Bitcoin series).

Comments

- **crypto** package offers historical data, starting from April 2013.
- For some relatively new cryptocurrencies, the size of data does not allow a consistent estimation of the Lyapunov exponent (defining the set of parameters as for the embedding dimension, time delay, number of neighbors).
- *Lyapunov exponent* is one tool to test the presence of chaotic patterns. Other measures may apply as for the *correlation dimension* or the *sample entropy*.
- In most financial time series, small fluctuations are persistent in contrast of large ones which are anti-persistent. The *Bitcoin* series demonstrates a persistent behavior of both fluctuations which cannot guarantee stability over time, making the process prone to potential crashes.

Conclusion

- Distinguishing the nature of the time series is crucial before analyzing/modeling data.
- Maximum Lyapunov exponents of the considered cryptocurrencies are positive : the presence of chaotic patterns is evident.
- The series exhibit a strong persistent behavior. Increments have a significant positive correlation (long-memory effect) which generate an unstable process prone to crashes.
- These findings reinforce the affirmation that prices of cryptocurrencies are unpredictable, risky and hard to model, compared to other traditional financial assets.

References

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- B. Mandelbrot, R.L. Hudson: The Misbehavior of Markets: A Fractal View of Financial Turbulence. Basic Books (2004).
- E.A. Ihlen: Introduction to multifractal detrended fluctuation analysis in Matlab. Frontiers in Physiology (2012) `https://doi.org/10.3389/fphys.2012.00141`
- R packages used: **dplyr**, **ggplot2**, **crypto**, **pracma**, **MFDFA**, **fNonlinear**, **tseriesChaos**, **nonlinearTseries**

Thank You for Your Attention !